# ON BAROCLINIC INSTABILITY AS A FUNCTION OF THE VERTICAL PROFILE OF THE ZONAL WIND [1]

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ABSTRACT

The quasi-geostrophic, baroclinic stability problem is solved for an arbitrary zonal wind profile $U=U(p)$ and for an adiabatic lapse rate. It is shown that the phase speed of the waves in this case depends on the vertical integrals of $U$ and $U^2$. Due to the assumption of an adiabatic stratification there is no short wave cutoff, but the effect of the variation of the Coriolis parameter will in all cases give stability for sufficiently long waves.

A number of numerical examples show that the region of instability, in a coordinate system with wavelength as abscissa and wind shear as ordinate, is the largest when the wind maximum is situated in the upper part of the atmosphere, and when the curvature of the zonal wind profile at the wind maximum has an intermediate value.

## 1. INTRODUCTION

Any solution of the baroclinic instability problem (Charney [2], Kuo [7]) has normally involved the assumption that the variation of the basic zonal wind with height or pressure is linear. Such assumptions have clearly been introduced for mathematical convenience. The general baroclinic instability problem can not be solved in closed analytical form for an arbitrary variation of the zonal wind with height. Some insight into the importance of the wind profile for the degree of instability has been obtained by Haltiner [6] using numerical methods. Another approach was used by Pedlosky [8] who found the changes in stability as a function of the velocity profile. He considered small arbitrary deviations from a uniform vertical wind shear. The starting point is Eady's [4] model, which considers the stability to quasi-geostrophic disturbances with a uniform vertical wind shear. In addition the Coriolis parameter is assumed constant. It is well known that one of the major effects of the variation of the Coriolis parameter with latitude (the $\beta$-effect) is to stabilize the very long waves.

A disadvantage of the numerical approach is that the phase velocity is not expressed explicitly in terms of the meteorological parameters entering the problem. The approach used by Pedlosky [8] requires that the deviations from a linear profile with respect to height are small. It is therefore of interest to consider still another approach to the problem. Considerable insight into the baroclinic stability problem was gained by considering a basic state with an adiabatic lapse rate (Fjørtoft [5]). Assuming that the variation of the zonal wind with height is linear, one can for example use this model to demonstrate the stabilizing effect of the $\beta$-term on the very long waves. A disadvantage of the model is that it predicts that very short waves are highly unstable. It is known from studies by Eady [4] and Fjørtoft [5] that a stable temperature structure with a lapse rate smaller than the adiabatic rate has a stabilizing effect on very short waves, and that non-geostrophic effects are important in this region. Any results which we may derive from a quasi-geostrophic theory, in particular, a model with an adiabatic stratification, are therefore unrealistic for short wavelengths and should be disregarded in this region. Comparative studies carried out by Arnason [1] and by Derome and Wiin-Nielsen [3] show, on the other hand, that the temperature structure in the basic state has relatively little influence on the stability properties for medium and long waves.

The main purpose of this study is to investigate the importance of the vertical variation of the zonal wind for the stability of unstable baroclinic disturbances. The study will be carried out by using a quasi-geostrophic model. The basic current will be specified as an arbitrary function of pressure, while the lapse rate in the basic state will be adiabatic. The last assumption combined with the fact that the theory is quasi-geostrophic means that the results will be applicable to long waves only.

## 2. SOLUTION OF THE PERTURBATION PROBLEM

The perturbation equations for the quasi-geostrophic problem have been derived several places in the literature. Using pressure as the vertical coordinate, we may write the basic non-linear equations in the form

$$\frac{\partial \zeta}{\partial t}+\mathbf{v}\cdot\nabla(\zeta+f)=f_0\frac{\partial \omega}{\partial p} \tag{2.1}$$

[1] Research supported by the National Science Foundation under Grant GP-2561. Publication No. 125 from the Department of Meteorology and Oceanography, University of Michigan, Ann Arbor, Mich.

$$\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial p}\right)+\mathbf{v}\cdot\nabla\left(\frac{\partial \psi}{\partial p}\right)+\frac{\sigma}{f_0}\omega=0 \tag{2.2}$$

in which $\zeta=\nabla^2\psi$ is the relative vorticity, $\psi$ the stream function, $\mathbf{v}$ the horizontal, non-divergent wind, $\omega$ the vertical velocity, $f$ the Coriolis parameter, $f_0$ a standard value, $\sigma=-(\alpha/\theta)(\partial\theta/\partial p)$ a measure of static stability, $\alpha$ the specific volume and $\theta$ the potential temperature.

The basic state is defined by a zonal wind $U=U(p)$ and a static stability $\bar{\sigma}=\bar{\sigma}(p)$ when we want to consider the purely baroclinic problem. The perturbations may be expressed in the form

$$\psi'(x, p, t)=\Psi(p)e^{ik(x-ct)} \tag{2.3}$$

$$\omega'(x, p, t)=\Omega(p)e^{ik(x-ct)} \tag{2.4}$$

in which we have neglected the dependence on the meridional coordinate.

When we linearize equations (2.1) and (2.2), introduce (2.3) and (2.4) in the linearized equations, and eliminate $\Psi=\Psi(p)$ from the resulting equations we obtain the following equations for $\Omega=\Omega(p)$:

$$(U-c)(U-c-c_R)\frac{d^2\Omega}{dp_*^2}-(2(U-c)-c_R)\frac{dU}{dp_*}\frac{d\Omega}{dv_*}$$

$$-\frac{k^2\bar{\sigma}p_0^2}{f_0^2}(U-c-c_R)^2\Omega=0 \tag{2.5}$$

in which we have introduced $c_R=\beta/k^2$ and the non-dimensional independent variable $p_*=p/p_0$, where $p_0$ is a standard value of the surface pressure ($p_0=100$ cb.). Equation (2.5) together with boundary conditions: $\Omega=0$ for $p_*=0$ and $p_*=1$ constitutes the general eigenvalue problem for $\Omega$ and the phase speed $c$.

The eigenvalue problem as stated above is very difficult to solve for an arbitrary specification of $U=U(p_*)$ and $\bar{\sigma}=\bar{\sigma}(p_*)$. Solutions to (2.5) have been obtained in several special cases all of which include the assumption that $U$ is a linear function of $p$. For a summary of these solutions the reader is referred to Derome and Wiin-Nielsen [3]. It is the purpose of this section to show that a complete solution can be obtained in the special case in which $\bar{\sigma}=0$. This case corresponds to an adiabatic lapse rate as can be seen from the definition of $\sigma$ given above. When $\bar{\sigma}=0$ we obtain the following equation:

$$E(E-c_R)\frac{d^2\Omega}{dp_*^2}-(2E-c_R)\frac{dE}{dp_*}\frac{d\Omega}{dp_*}=0 \tag{2.6}$$

in which $E=E(p)=U-c$.

We note that (2.6) can be written in the form:

$$E^2(E-c_R)^2\frac{d}{dp_*}\left[\frac{d\Omega/dp_*}{E(E-c_R)}\right]=0. \tag{2.7}$$

From (2.7) we find that

$$\frac{d\Omega}{dp_*}=AE(E-c_R) \tag{2.8}$$

where $A$ is an integration constant. (2.8) can be integrated directly to give

$$\Omega=A\int_0^{p_*}E(E-c_R)dp_* \tag{2.9}$$

which is the general solution to (2.6).

The eigenvalue $c$, the phase speed, is determined from the boundary condition that $\Omega=0$ for $p_*=1$. We get therefore

$$\int_0^1 E(E-c_R)dp_*=0. \tag{2.10}$$

When we introduce $E=U-c$ in (2.10) and carry out the integrations we get the following equations for the phase speed:

$$c^2-(2I_1-c_R)c+(I_2-c_RI_1)=0 \tag{2.11}$$

in which $I_1$ and $I_2$ are notations for the integrals:

$$I_1=\int_0^1 Udp_* \tag{2.12}$$

and

$$I_2=\int_0^1 U^2dp_*. \tag{2.13}$$

We obtain an important conclusion directly from (2.11). It is seen that the phase speed depends on the integrals (2.12) and (2.13), but not on the detailed structure of $U=U(p_*)$. We note furthermore that $I_1$ is identical to the vertical average of $U(p_*)$, while $I_2$ is the vertical average of the function $U^2$. For later use we note that the following inequality holds:

$$I_2\geq I_1^2. \tag{2.14}$$

The relation (2.14) is a special case of Schwartz's inequality, but the proof is easily obtained in this case by noting that we may write $U$ in the form

$$U=I_1+U' \tag{2.15}$$

where $I_1$ is the vertical mean value and $U'$ is the deviation from this mean value. We note, in particular, that $\int_0^1 U'dp_*=0$. Using (2.15), we obtain

$$I_2=\int_0^1(I_1^2+U'^2+2I_1U')dp_*=I_1^2+\int_0^1 U'^2dp_* \tag{2.16}$$

from which we obtain (2.14). It is seen that the equality sign holds only when $U'=0$ everywhere, i.e. when $U=$constant.

The solution to (2.11) is

$$c=(I_1-\tfrac{1}{2}c_R)\pm\sqrt{I_1^2-I_2+\tfrac{1}{4}c_R^2}. \tag{2.17}$$

It is seen from (2.17) that if we neglect the $\beta$-effect, i.e. $c_R=0$, there will always be unstable waves because of (2.14). They will move with the speed $I_1$ while the imaginary part of the phase speed will be $c_i=(I_2-I_1^2)^{1/2}$. Equation (2.17) shows clearly the stabilizing effect of $\beta$ on the very long waves. For a sufficiently small wave number $c_R$ will be so large that the radicand in (2.17) will become positive giving stability. The general nature of the stability diagram obtained for the case when $U$ is a linear function of $p_*$ is therefore unchanged, although there will be changes in details.

It should be pointed out that (2.17) can be written in the form

$$c=(I_1-\tfrac{1}{2}c_R)\pm\sqrt{\tfrac{1}{4}c_R^2-I_3} \tag{2.18}$$

where

$$I_3=I_2-I_1^2=\int_0^1 U'^2 dp_*. \tag{2.19}$$

The last expression in (2.19) is obtained by the use of (2.16). It appears therefore that $I_3$ is the important integral for the determination of the imaginary part of the phase speed, while $I_1$ determines the real part. The baroclinic instability in the present model is therefore determined by the square of the variance of the zonal flow from its vertical mean value $I_1$.

The general solution given in (2.17) and (2.18) can be compared with other models for which we have a complete solution. The comparison will be made with the well-known stability analysis of the quasi-geostrophic, two-level model as given for example by Thompson [9]. He finds

$$c=\overline{U}-\frac{1}{2}c_R\frac{2k^2+\mu^2}{k^2+\mu^2}\pm\sqrt{D} \tag{2.20}$$

where

$$D=\frac{1}{4}c_R^2\frac{\mu^4}{(k^2+\mu^2)^2}-U_T^2\frac{\mu^2-k^2}{\mu^2+k^2} \tag{2.21}$$

in which $\overline{U}=\frac{1}{2}(U_1+U_3)$ and $U_T=\frac{1}{2}(U_1-U_3)$, where subscripts 1 and 3 refer to the 250-mb. and 750-mb. levels, respectively. $\mu^2$ is a parameter which is inversely proportional to the static stability $\sigma$. When $\mu^2\rightarrow\infty$, which corresponds to the two-level analogue of the model investigated in this paper, we find

$$c=U-\tfrac{1}{2}c_R\pm\sqrt{\tfrac{1}{4}c_R^2-U_T^2}. \tag{2.22}$$

$\overline{U}$ is the vertical mean of the basic flow and corresponds therefore to $I_1$. $U_T^2$, on the other hand, is closely related to $I_3$. If we for example assume that the zonal wind in the two-level model varies linearly with pressure we find

$$U=\overline{U}+2U_T(1-2p_*) \tag{2.23}$$

from which it follows that $I_3=(4/3)U_T^2$. If we on the other hand assume that the zonal wind in the layer 0 to 50 cb. is constant and equal to $U_1$, while the wind in the layer 50 to 100 cb. is constant and equal to $U_3$, we find that $I_3=U_T^2$. It is in any case evident that the stability analysis of the two-level model corresponds very closely to our general result when $\mu^2\rightarrow\infty$.

The advantage of the comparison is that it seems likely that the zonal wind profile which gives the greatest region of instability in our model would also be the most unstable in a model with a stratification different from the adiabatic situation employed here.

## 3. SOME NUMERICAL EXAMPLES

In this section we shall investigate various classes of wind profiles in order to find how the stability properties change as a function of the profile.

The first example will be selected in such a way that we may investigate the importance of the position of the wind maximum. For this purpose we have selected a function $U=U(p_*)$ as follows:

$$U=BU_m p_*(1-p_*)^q. \tag{3.1}$$

By differentiation with respect to $p_*$ we find that $dU/dp_*=0$ if $p_*=1$ and $p_*=(q+1)^{-1}$. Selecting the second of these values we find that the maximum value of $U$ is:

$$U_{max}=BU_m\frac{1}{q+1}\left(\frac{q}{q+1}\right)^q. \tag{3.2}$$

If we therefore select $B$ to have the value

$$B=(q+1)^{q+1}/q^q \tag{3.3}$$

we find that $U_{max}=U_m$. The non-dimensional pressure for which the maximum occurs is

$$p_{*,m}=1/(q+1) \tag{3.4}$$

and it is therefore seen that large values of $q$ imply a wind maximum for a small value of $p_*$. Figure 1 shows the wind profiles $U/U_m$ as a function of $p_*$ for various values of $q$.

In investigating the stability we shall be satisfied by finding the critical curve which according to (2.17) is given by the expression

$$I_2-I_1^2=\tfrac{1}{4}c_R^2. \tag{3.5}$$

The integrals $I_1$ and $I_2$ can be evaluated from the definitions by elementary integrations. We find:

$$I_1=U_m\frac{(q+1)^{q+1}}{q^q}\frac{1}{(q+1)(q+2)} \tag{3.6}$$

and

$$I_2=U_m^2\frac{(q+1)^{2q+2}}{q^{2q}}\frac{2}{(2q+1)(2q+2)(2q+3)} \tag{3.7}$$

When $I_1$ and $I_2$ are substituted in (3.5), we can solve for $U_m$. In evaluating the formula we substitute $c_R=\beta/k^2=0.4l^2$, where $l$ is the wavelength measured in $10^6$ m. and $\beta=16\times10^{-12}$ m.$^{-1}$ sec.$^{-1}$ The value of $U_m$ is a measure of the wind shear in the lower part of the atmosphere, but $U_m$ occurs at various pressures for different values of $q$. We define therefore a wind shear $U_s$ as follows:

$$U_s=\frac{1}{1-p_{*,m}}\int_{p_{*,m}}^1\frac{dU}{dp_*}dp_*=-\frac{q+1}{q}U_m \tag{3.8}$$

where $p_{*,m}$ is given by (3.4). Solving for $U_s$, we get

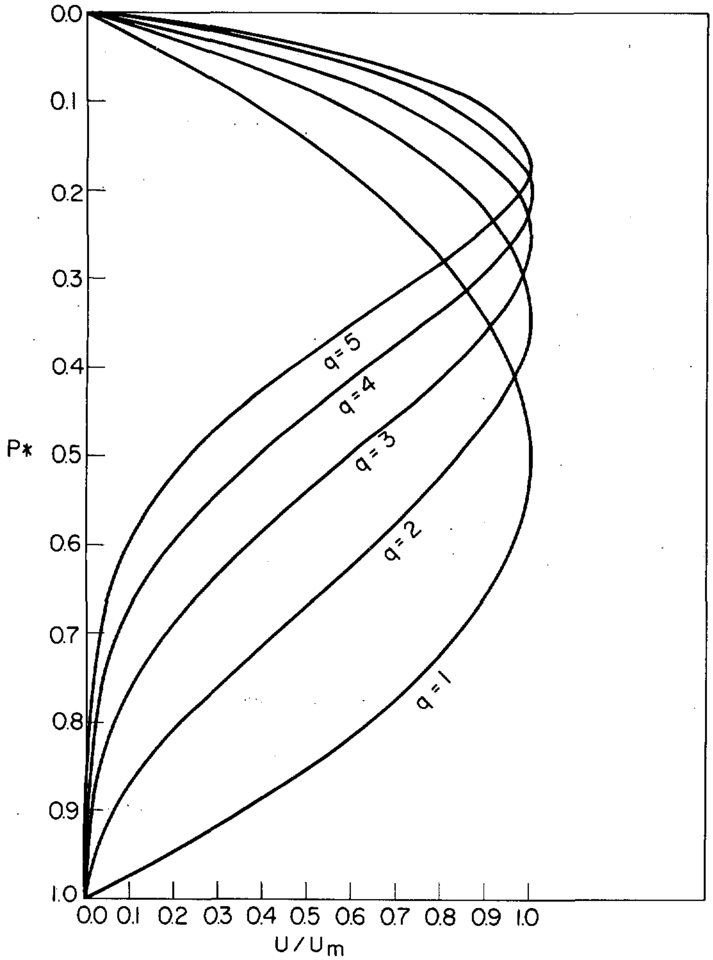


FIGURE 1.—The zonal wind as a function of pressure for various values of $q$ based on equation (3.1).

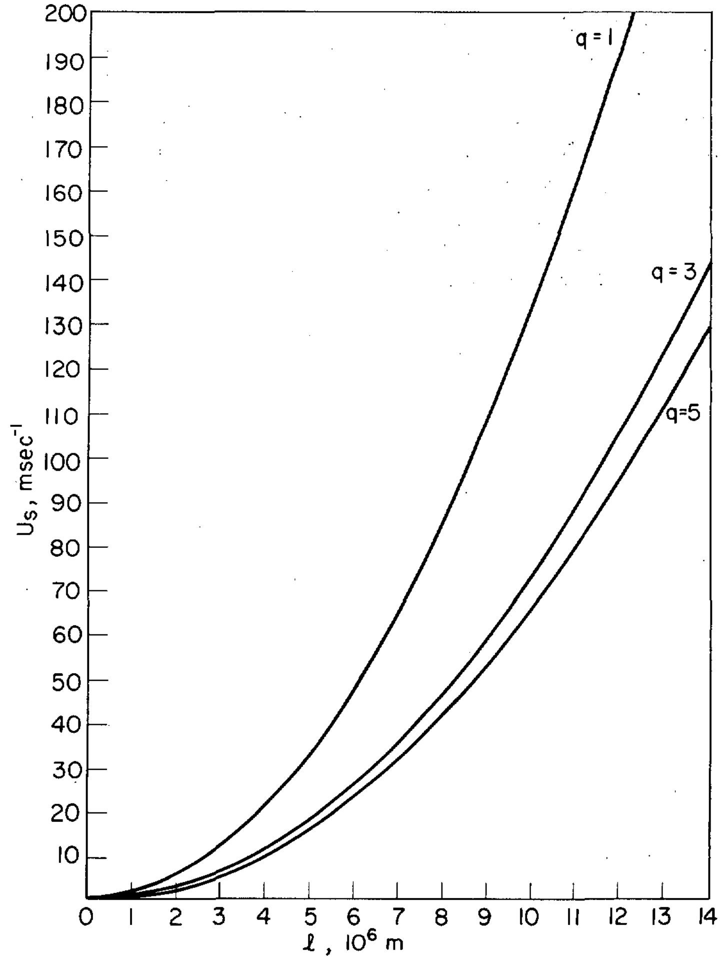


FIGURE 2.—Curves of neutral stability for various values of $q$ in a diagram with wavelength ($10^6$m.) as abscissa and the vertical wind shear, $U_s$(m. sec.$^{-1}$), as ordinate.

$$U_s=C_q l^2 \tag{3.9}$$

in which

$$C_q=0.2\,\frac{q^{q-1}}{(q+1)^q}\cdot\frac{1}{\sqrt{s}} \tag{3.10}$$

and

$$s=\frac{2}{(2q+1)(2q+2)(2q+3)}-\frac{1}{(q+1)^2(q+2)^2}. \tag{3.11}$$

The critical curve is therefore in all cases a parabola in a diagram with $l$ as abscissa and $U_s$ as ordinate. The region of instability is above the parabola. The coefficient $C_q$ is a measure of the region of instability. The first few values of $C_q$ are given in table 1. It is seen that the smaller values of $q$ correspond to the larger values of $C_q$ and therefore to a smaller region of instability. It is instructive to compare these results with the region of instability for a wind profile $U=U_m p_*(q=0)$ or $U=U_m(1-p_*)$ in which cases we find $U_s=0.68\ l^2$. The linear wind profile has therefore about the same region of instability as the case $q=4$ in table 1. We note next that there is a minimum in $C_q$ around $q=7$ which indicates that the greatest region of instability occurs when the wind maximum is located around $p_{*,m}=0.125$, a result which may be modified if we were able to consider the influence of static stability. The regions of instability are given in figure 2 for selected values of $q$. We note in the figure that the curve $q=5$ for practical purposes is identical to the maximum region of instability.

The wind profiles described by (3.1) will have the maximum wind at a value of $p_*\leq½$. From this point of view they approximate atmospheric conditions. One may, however, ask how the stability is changed if the maximum occurs at $p_*\geq½$. This question can be answered by a consideration of the wind profiles:

$$U=DU_m p_*^q(1-p_*),\qquad q\geq 1. \tag{3.12}$$

TABLE 1.—*The coefficient* $C_q$ *as a function of* q

| $q$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_q$ | 1.34 | 0.88 | 0.73 | 0.68 | 0.66 | 0.65 | 0.65 | 0.66 | 0.67 | 0.67 |

By differentiation with respect to $p_*$ it is found that the maximum is at $p_{*,m}=q/(q+1)$ and that the maximum value $U_{max}=U_m$ if $D=(q+1)^{q+1}/q^q$. An evaluation of $I_1$ and $I_2$ leads to the same values as given in (3.6) and (3.7). However, the value of $U_s$ as given by (3.8) is now $U_s=-(q+1)U_m$ and equivalent to (3.9) we find now

$$U_s=C_q^*\cdot l^2 \tag{3.13}$$

where

$$C_q^*=0.2\,\frac{q^q}{(q+1)^q}\cdot\frac{1}{\sqrt{s}}. \tag{3.14}$$

A comparison of (3.10) and (3.14) shows that

$$C_q^*=q\cdot C_q. \tag{3.15}$$

Using the values of table 1 we find that the region of instability in a diagram corresponding to figure 1 will be even smaller when we use (3.12) in place of (3.1). The first example tested the stability as a function of the position of the wind maximum in the basic zonal current. In our next example we shall keep the wind maximum at the same pressure level but vary the shape of the profile. For this purpose we select the following wind profile

$$U=U_m\,\frac{p_*^\alpha(1-p_*)^\beta}{p_m^\alpha(1-p_m)^\beta} \tag{3.16}$$

in which $p_m$ and $U_m$ are constants. Differentiating (3.16) with respect to $p_*$ we obtain

$$\frac{dU}{dp_*}=\frac{U_m}{p_m^\alpha(1-p_m)^\beta}\,p_*^{\alpha-1}(1-p_*)^{\beta-1}[\alpha(1-p_*)-\beta p_*]. \tag{3.17}$$

The zero points of (3.17) are $p_*=0$ and $p_*=1$ if $\alpha\geq 1$ and $\beta\geq 1$. An additional root is $p_*=\alpha/(\alpha+\beta)$. We want to select $\alpha$ and $\beta$ in such a way that the last root occurs for $p_*=p_m$ which gives the following condition

$$\beta=\left(\frac{1-p_m}{p_m}\right)\alpha. \tag{3.18}$$

Let us select $p_m=\frac{1}{4}$ which means that the maximum wind will be found at 25 cb. We then get $\beta=3\alpha$.

The integrals $I_1$ and $I_2$ can be evaluated by repeated integrations by parts. We find:

$$I_1=\frac{\alpha!\beta!}{(\alpha+\beta+1)!}\,\frac{U_m}{p_m^\alpha(1-p_m)^\beta} \tag{3.19}$$

and

$$I_2=\frac{(2\alpha)!(2\beta)!}{(2\alpha+2\beta+1)!}\,\frac{U_m^2}{p_m^{2\alpha}(1-p_m)^{2\beta}}. \tag{3.20}$$

Substituting (3.19) and (3.20) in the equation (3.5) for the neutral curve, we find again an equation of the form (3.9), but the coefficient is now given by the following expression

$$C_\alpha=0.2p_m^\alpha(1-p_m)^{3\alpha}\cdot s(\alpha)^{-1/2} \tag{3.21}$$

where

$$s(\alpha)=\frac{(2\alpha)!(2\beta)!}{(2\alpha+\alpha\beta+1)!}-\left[\frac{\alpha!\beta!}{(\alpha+\beta+1)!}\right]^2. \tag{3.22}$$

Figure 3 shows the curves $U/U_m$ computed from (3.16) with $p_m=\frac{1}{4}$ for $\alpha=1$, 2, and 3. It is seen that the larger values of $\alpha$ correspond to a sharper wind maximum. The stability of the zonal currents will be investigated and compared with the straight line profile described by

$$U=\begin{cases} U_m\,\dfrac{p_*}{p_m}, & 0\leq p_*\leq p_m \\ U_m\,\dfrac{1-p_*}{1-p_m}, & p_m\leq p_*\leq 1. \end{cases} \tag{3.23}$$

Using (3.23) we find $I_1=\frac{1}{2}U_m$ and $I_2=\frac{1}{3}U_m^2$. In this case we find the neutral curve to be $U_m=0.4\sqrt{3}\,l^2$, corresponding to a value of $C_\alpha=0.68$.

The coefficient $C_\alpha$ given by (3.21) has been evaluated for $p_m=\frac{1}{4}$ and for $\alpha=1, 2, 3, \ldots, 10$ in order to find out if there is a profile in the class given by (3.16) which shows a larger area of instability. The results of this calculation

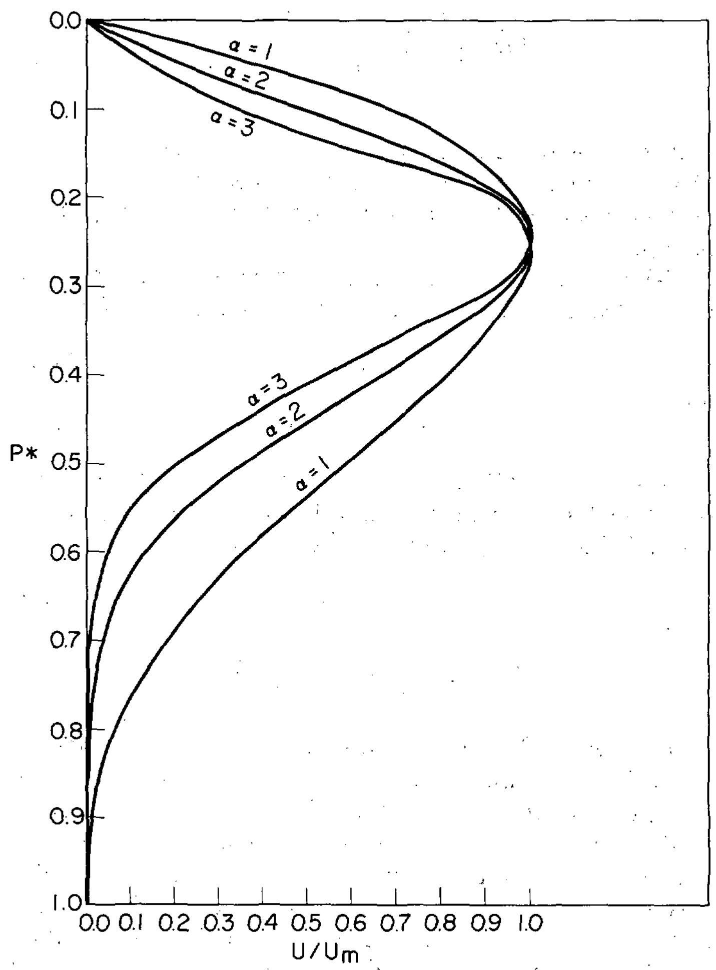


Figure 3.—The zonal wind as a function of pressure for various values of $\alpha$ based on equation (3.12). The curves show a wind maximum at 25 cb. ($\beta=3\alpha$.)

TABLE 2.—*The coefficient $C_\alpha$ as a function of $\alpha$*

| $\alpha$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_\alpha$ | 0.55 | 0.54 | 0.56 | 0.58 | 0.59 | 0.61 | 0.62 | 0.64 | 0.65 | 0.66 |

are given in table 2 in which $C_\alpha$ is given as a function of $\alpha$. We notice from the table that the minimum value of $C_\alpha$ occurs for $\alpha=2$, which means that the maximum region of instability is found for this value.

The main result from the first two examples is that the maximum region of instability is found when the wind maximum is located at relatively low values of pressure (12.5 cb.), and when the profile has a well-defined maximum, but not too sharp a peak. We may test the second part of this tentative conclusion by considering a third example in which we consider a profile which is continuous, but where the first derivative with respect to pressure is discontinuous at the wind maximum.

The zonal wind in the third example is given by the following expressions:

$$U(p_*)=\begin{cases} U_m \dfrac{p_*^s}{p_m^s}, & 0\leq p_*\leq p_m \\ U_m \dfrac{(1-p_*)^s}{(1-p_m)^s}, & p_m\leq p_*\leq 1. \end{cases} \quad (3.24)$$

A few representative wind profiles for various values of $s$ are given in figure 4. It is seen that the larger values of $s$ correspond to a larger discontinuity in $dU/dp_*$ at $p_*=p_m$.

Using (3.24), we may next calculate the integrals $I_1$ and $I_2$ from (2.12) and (2.13), respectively. We find

$$I_1=U_m/(s+1) \quad (3.25)$$

and

$$I_2=U_m^2/(2s+1). \quad (3.26)$$

Equations (3.25) and (3.26) are substituted in (3.5), the equation for the neutral stability curve. After reduction we find that the equation for this curve is:

$$U_m=C_s l^2 \quad (3.27)$$

in which

$$C_s=0.2\left(\frac{1}{2s+1}-\frac{1}{(s+1)^2}\right)^{-1/2}$$

In order to find the maximum region of instability we must find a minimum value of $C_s$ as a function of $s$. A straightforward differentiation of $C_s$ with respect to $s$ leads to the result that $C_s$ has a minimum when $s=\frac{1}{2}(1+\sqrt{5})=1.618$. The minimum value of $C_s$ is $C_s=0.666$. The variation of $C_s$ as a function of $s$ is given in table 3 ($0\leq s\leq 1$) and table 4 ($1\leq s\leq 10$). The results listed in these tables confirm the tentative conclusion reached in our second example. The curve corresponding to $s=2$ in figure 4 is a close approximation of the curve ($s=\frac{1}{2}(1+\sqrt{5})$) which has the largest region of instability.

TABLE 3.—*The coefficient $C_s$ as a function of $s$ ($0\leq s\leq 1$)*

| $s$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_s$ | 2.41 | 1.42 | 1.10 | 0.94 | 0.85 | 0.79 | 0.75 | 0.73 | 0.71 | 0.69 |

TABLE 4.—*The coefficient $C_s$ as a function of $s$ ($1\leq s\leq 10$)*

| $s$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_s$ | 0.69 | 0.67 | 0.71 | 0.75 | 0.80 | 0.84 | 0.89 | 0.93 | 0.97 | 1.01 |

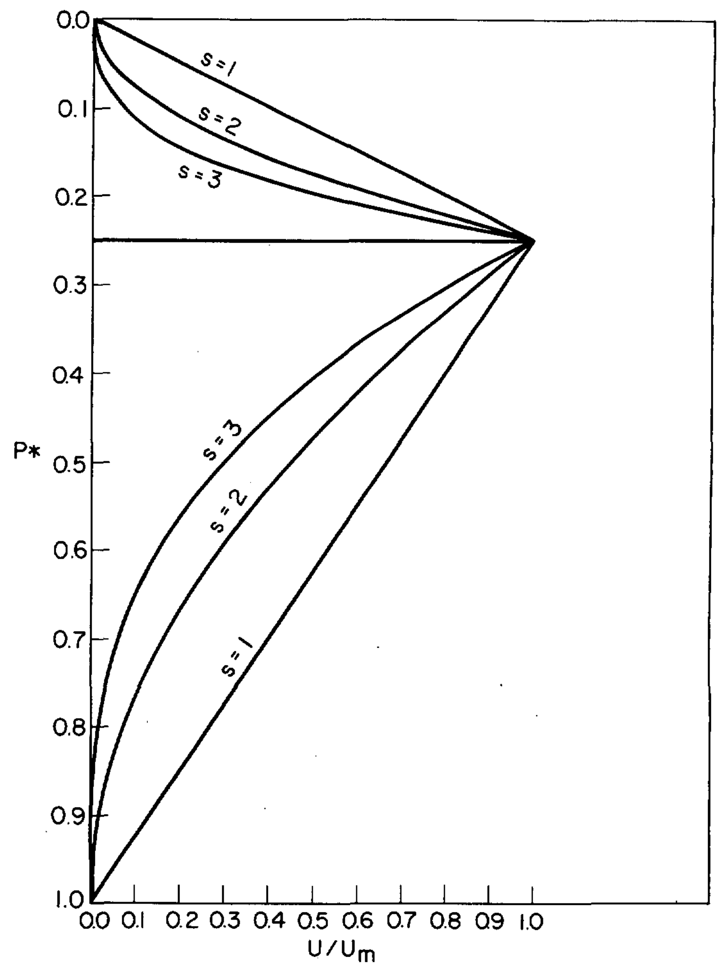


FIGURE 4.—The zonal wind as a function of pressure for various values of $s$ based on equation (3.20).

## 4. CONCLUDING REMARKS

The main purpose of this paper is to investigate the influence of the shape of the zonal wind profile on the instability of atmospheric disturbances. In order to obtain a solution in closed form it has been necessary to use a quasi-geostrophic formulation and to make the additional assumption that the temperature stratification is adiabatic. The latter assumption limits the applicability to long waves, but makes it possible to obtain a solution for an arbitrary wind profile $U=U(p)$. This solution is investigated in general and several numerical examples are given. The main conclusions are that the maximum region of instability is found when the wind maximum occurs in the higher parts of the atmosphere, and when the curvature at the wind maximum is of a moderate magnitude.

### REFERENCES

1. G. 'Arnason, "The Stability of Non-Geostrophic Perturbations in a Baroclinic Zonal Flow," *Tellus*, vol. 15, No. 3, Aug. 1963, pp. 205–209.
2. J. G. Charney, "The Dynamics of Long Waves in a Baroclinic Westerly Current," *Journal of Meteorology*, vol. 4, No. 5, Oct. 1947, pp. 135–162.
3. J. F. Derome and A. Wiin-Nielsen, "On the Baroclinic Stability of Zonal Flow in Simple Model Atmospheres," *Technical Report* No. 2 (06372–2–T), University of Michigan, Oct. 1966, 93 pp.
4. E. T. Eady, "Long Waves and Cyclone Waves," *Tellus*, vol. 1, No. 3, Aug. 1949, pp. 33–52.
5. R. Fjørtoft, "Application of Integral Theorems in Deriving Criteria of Stability for Laminar Flows and for the Baroclinic Circular Vortex," *Geofysiske Publikasjoner*, vol. 17, No. 6, Mar. 1950, pp. 1–52.
6. G. J. Haltiner, "Finite Difference Approximations for the Determination of Dynamic Instability," *Tellus*, vol. 15, No. 3, Aug. 1963, pp. 230–240.
7. H. L. Kuo, "Three-Dimensional Disturbances in a Baroclinic Zonal Current," *Journal of Meteorology*, vol. 9, No. 4, Aug. 1952, pp. 260–278.
8. J. Pedlosky, "On the Stability of Baroclinic Flows as a Functional of the Velocity Profile," *Journal of Atmospheric Sciences*, vol. 22, No. 2, Mar. 1965, pp. 137–145.
9. P. D. Thompson, *Numerical Weather Analysis and Prediction*, Macmillan Company, New York, 1961, 170 pp.

*[Received June 26, 1967; revised August 17, 1967]*